

In[1]:= Integrate[(1/L) * ((Cos[2 * Pi * t / L] + 1) / 2) * (E^((a - (I * 2 * Pi * f)) * t)),
{t, (-L/2), (L/2)}]

Out[1]=
$$\frac{4 \pi^2 \operatorname{Sinh}\left[\frac{1}{2} L (a - 2 i f \pi)\right]}{L (a - 2 i f \pi) (a^2 L^2 - 4 i a f L^2 \pi - 4 (-1 + f^2 L^2) \pi^2)}$$

In[2]:= Manipulate[

$$\left(4 \pi^2 \operatorname{Sinh}\left[\frac{1}{2} L (a - 2 i f \pi)\right]\right) / (L (a - 2 i f \pi) (a^2 L^2 - 4 i a f L^2 \pi - 4 (-1 + f^2 L^2) \pi^2)), \{L, \{1\}\}]$$

Out[2]=

L

$$\frac{4 \pi^2 \operatorname{Sinh}\left[\frac{1}{2} (a - 2 i f \pi)\right]}{(a - 2 i f \pi) (a^2 - 4 i a f \pi - 4 (-1 + f^2) \pi^2)}$$

$$\text{In[5]= ComplexExpand} \left[\frac{4 \pi^2 \text{Sinh} \left[\frac{1}{2} (a - 2 i f \pi) \right]}{(a - 2 i f \pi) (a^2 - 4 i a f \pi - 4 (-1 + f^2) \pi^2)} \right]$$

$$\begin{aligned} \text{Out[5]=} & \frac{24 a^2 f \pi^3 \text{Cosh} \left[\frac{a}{2} \right] \text{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2)} + \frac{32 f \pi^5 \text{Cosh} \left[\frac{a}{2} \right] \text{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2)} - \\ & \frac{32 f^3 \pi^5 \text{Cosh} \left[\frac{a}{2} \right] \text{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2)} + \\ & \frac{4 a^3 \pi^2 \text{Cos}[f \pi] \text{Sinh} \left[\frac{a}{2} \right]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2)} + \\ & \frac{16 a \pi^4 \text{Cos}[f \pi] \text{Sinh} \left[\frac{a}{2} \right]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2)} - \\ & \frac{48 a f^2 \pi^4 \text{Cos}[f \pi] \text{Sinh} \left[\frac{a}{2} \right]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2)} + \\ & i \left(- \frac{4 a^3 \pi^2 \text{Cosh} \left[\frac{a}{2} \right] \text{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2)} - \right. \\ & \frac{16 a \pi^4 \text{Cosh} \left[\frac{a}{2} \right] \text{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2)} + \\ & \frac{48 a f^2 \pi^4 \text{Cosh} \left[\frac{a}{2} \right] \text{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2)} + \\ & \frac{24 a^2 f \pi^3 \text{Cos}[f \pi] \text{Sinh} \left[\frac{a}{2} \right]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2)} + \\ & \frac{32 f \pi^5 \text{Cos}[f \pi] \text{Sinh} \left[\frac{a}{2} \right]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2)} - \\ & \left. \frac{32 f^3 \pi^5 \text{Cos}[f \pi] \text{Sinh} \left[\frac{a}{2} \right]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2)} \right) \end{aligned}$$

$$\text{In[6]:= } D\left[\text{ArcTan}\left[\begin{aligned} & -\frac{4 a^3 \pi^2 \text{Cosh}\left[\frac{a}{2}\right] \text{Sin}[f \pi]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} - \\ & \frac{16 a \pi^4 \text{Cosh}\left[\frac{a}{2}\right] \text{Sin}[f \pi]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} + \\ & \frac{48 a f^2 \pi^4 \text{Cosh}\left[\frac{a}{2}\right] \text{Sin}[f \pi]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} + \\ & \frac{24 a^2 f \pi^3 \text{Cos}[f \pi] \text{Sinh}\left[\frac{a}{2}\right]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} + \\ & \frac{32 f \pi^5 \text{Cos}[f \pi] \text{Sinh}\left[\frac{a}{2}\right]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} - \\ & \frac{32 f^3 \pi^5 \text{Cos}[f \pi] \text{Sinh}\left[\frac{a}{2}\right]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} \right] / \\ & \left(\frac{24 a^2 f \pi^3 \text{Cosh}\left[\frac{a}{2}\right] \text{Sin}[f \pi]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} + \right. \\ & \frac{32 f \pi^5 \text{Cosh}\left[\frac{a}{2}\right] \text{Sin}[f \pi]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} - \\ & \frac{32 f^3 \pi^5 \text{Cosh}\left[\frac{a}{2}\right] \text{Sin}[f \pi]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} + \\ & \frac{4 a^3 \pi^2 \text{Cos}[f \pi] \text{Sinh}\left[\frac{a}{2}\right]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} + \\ & \left. \frac{16 a \pi^4 \text{Cos}[f \pi] \text{Sinh}\left[\frac{a}{2}\right]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} - \right. \\ & \left. \frac{48 a f^2 \pi^4 \text{Cos}[f \pi] \text{Sinh}\left[\frac{a}{2}\right]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)}\right], f] \end{aligned}$$

$$\text{Out[6]= } \left(-\left(\left(-\frac{4 a^3 \pi^2 \text{Cosh}\left[\frac{a}{2}\right] \text{Sin}[f \pi]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} - \right.\right.\right. \\ \frac{16 a \pi^4 \text{Cosh}\left[\frac{a}{2}\right] \text{Sin}[f \pi]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} + \\ \frac{48 a f^2 \pi^4 \text{Cosh}\left[\frac{a}{2}\right] \text{Sin}[f \pi]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} + \\ \left.\left.\left.\frac{24 a^2 f \pi^3 \text{Cos}[f \pi] \text{Sinh}\left[\frac{a}{2}\right]}{\left(a^2+4 f^2 \pi^2\right)\left(16 a^2 f^2 \pi^2+\left(a^2-4(-1+f^2) \pi^2\right)^2\right)} + \right.\right.\right.$$

$$\begin{aligned}
& \left(\frac{32 f \pi^5 \cos[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \right. \\
& \left. \frac{32 f^3 \pi^5 \cos[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} \right) \\
& \left(\frac{24 a^2 f \pi^4 \cos[f \pi] \operatorname{Cosh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \right. \\
& \frac{32 f \pi^6 \cos[f \pi] \operatorname{Cosh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \frac{32 f^3 \pi^6 \cos[f \pi] \operatorname{Cosh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \frac{24 a^2 f \pi^3 \left(32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)\right) \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)^2} - \\
& \frac{32 f \pi^5 \left(32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)\right) \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)^2} + \\
& \frac{32 f^3 \pi^5 \left(32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)\right) \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)^2} - \\
& \frac{192 a^2 f^2 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2)^2 \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \frac{256 f^2 \pi^7 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2)^2 \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{256 f^4 \pi^7 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2)^2 \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{24 a^2 \pi^3 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{32 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \frac{96 f^2 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \frac{4 a^3 \pi^2 \left(32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)\right) \cos[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)^2} - \\
& \frac{16 a \pi^4 \left(32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)\right) \cos[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)^2} +
\end{aligned}$$

$$\begin{aligned}
& \frac{48 a f^2 \pi^4 \left(32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4 (-1 + f^2) \pi^2) \right) \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)^2} - \\
& \frac{32 a^3 f \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right)^2 \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} - \\
& \frac{128 a f \pi^6 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right)^2 \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} + \\
& \frac{384 a f^3 \pi^6 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right)^2 \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} - \\
& \frac{96 a f \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} - \\
& \frac{4 a^3 \pi^3 \operatorname{Sin}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} - \\
& \frac{16 a \pi^5 \operatorname{Sin}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} + \\
& \frac{48 a f^2 \pi^5 \operatorname{Sin}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} \Bigg) \Bigg) / \\
& \left(\frac{24 a^2 f \pi^3 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} + \right. \\
& \frac{32 f \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} - \\
& \frac{32 f^3 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} + \\
& \frac{4 a^3 \pi^2 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} + \\
& \frac{16 a \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} - \\
& \left. \frac{48 a f^2 \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} \right)^2 + \\
& \left(- \frac{4 a^3 \pi^3 \operatorname{Cos}[f \pi] \operatorname{Cosh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} - \right. \\
& \left. \frac{16 a \pi^5 \operatorname{Cos}[f \pi] \operatorname{Cosh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2 \right)} + \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{48 a f^2 \pi^5 \cos[f \pi] \cosh\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{4 a^3 \pi^2 \left(32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)\right) \cosh\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)^2} + \\
& \frac{16 a \pi^4 \left(32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)\right) \cosh\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)^2} - \\
& \frac{48 a f^2 \pi^4 \left(32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)\right) \cosh\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)^2} + \\
& \frac{32 a^3 f \pi^4 \cosh\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2)^2 \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{128 a f \pi^6 \cosh\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2)^2 \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \frac{384 a f^3 \pi^6 \cosh\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2)^2 \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{96 a f \pi^4 \cosh\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \frac{24 a^2 f \pi^3 \left(32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)\right) \cos[f \pi] \sinh\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)^2} - \\
& \frac{32 f \pi^5 \left(32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)\right) \cos[f \pi] \sinh\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)^2} + \\
& \frac{32 f^3 \pi^5 \left(32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)\right) \cos[f \pi] \sinh\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)^2} - \\
& \frac{192 a^2 f^2 \pi^5 \cos[f \pi] \sinh\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2)^2 \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \frac{256 f^2 \pi^7 \cos[f \pi] \sinh\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2)^2 \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{256 f^4 \pi^7 \cos[f \pi] \sinh\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2)^2 \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{24 a^2 \pi^3 \cos[f \pi] \sinh\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{32 \pi^5 \cos[f \pi] \sinh\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} -
\end{aligned}$$

$$\begin{aligned}
& \frac{96 f^2 \pi^5 \cos[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \frac{24 a^2 f \pi^4 \sin[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \frac{32 f \pi^6 \sin[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{32 f^3 \pi^6 \sin[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} \Bigg) / \\
& \left(\frac{24 a^2 f \pi^3 \operatorname{Cosh}\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \right. \\
& \frac{32 f \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \frac{32 f^3 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{4 a^3 \pi^2 \cos[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{16 a \pi^4 \cos[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \left. \frac{48 a f^2 \pi^4 \cos[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} \right) / \\
& \left(1 + \left(- \frac{4 a^3 \pi^2 \operatorname{Cosh}\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \right. \right. \\
& \frac{16 a \pi^4 \operatorname{Cosh}\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{48 a f^2 \pi^4 \operatorname{Cosh}\left[\frac{a}{2}\right] \sin[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{24 a^2 f \pi^3 \cos[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} + \\
& \frac{32 f \pi^5 \cos[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} - \\
& \left. \left. \frac{32 f^3 \pi^5 \cos[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2\right)} \right)^2 \right) /
\end{aligned}$$

$$\left(\frac{24 a^2 f \pi^3 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} + \frac{32 f \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} - \frac{32 f^3 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} + \frac{4 a^3 \pi^2 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} + \frac{16 a \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} - \frac{48 a f^2 \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} \right)^2$$

$$\ln[7]:= \text{Manipulate} \left[\left(\left(\left(- \frac{4 a^3 \pi^2 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} - \frac{16 a \pi^4 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} + \frac{48 a f^2 \pi^4 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} + \frac{24 a^2 f \pi^3 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} + \frac{32 f \pi^5 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} - \frac{32 f^3 \pi^5 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} \right) \right) \right) \left(\frac{24 a^2 f \pi^4 \operatorname{Cos}[f \pi] \operatorname{Cosh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} + \frac{32 f \pi^6 \operatorname{Cos}[f \pi] \operatorname{Cosh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} - \frac{32 f^3 \pi^6 \operatorname{Cos}[f \pi] \operatorname{Cosh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right)} \right) \left(24 a^2 f \pi^3 (32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4 (-1 + f^2) \pi^2)) \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi] \right) / \left((a^2 + 4 f^2 \pi^2) \left(16 a^2 f^2 \pi^2 + (a^2 - 4 (-1 + f^2) \pi^2)^2\right) \right)^2 -$$

$$\begin{aligned}
& \left(32 f \pi^5 (32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)) \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi] \right) / \\
& \left((a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2) \right)^2 + \\
& \left(32 f^3 \pi^5 (32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)) \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi] \right) / \\
& \left((a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2) \right)^2 - \\
& \frac{192 a^2 f^2 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2)^2 (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2)} - \\
& \frac{256 f^2 \pi^7 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2)^2 (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2)} + \\
& \frac{256 f^4 \pi^7 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2)^2 (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2)} + \\
& \frac{24 a^2 \pi^3 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2)} + \\
& \frac{32 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2)} - \\
& \frac{96 f^2 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2)} - \\
& \left(4 a^3 \pi^2 (32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)) \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right] \right) / \\
& \left((a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2) \right)^2 - \\
& \left(16 a \pi^4 (32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)) \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right] \right) / \\
& \left((a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2) \right)^2 + \\
& \left(48 a f^2 \pi^4 (32 a^2 f \pi^2 - 16 f \pi^2 (a^2 - 4(-1 + f^2) \pi^2)) \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right] \right) / \\
& \left((a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2) \right)^2 - \\
& \frac{32 a^3 f \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2)^2 (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2)} - \\
& \frac{128 a f \pi^6 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2)^2 (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2)} + \\
& \frac{384 a f^3 \pi^6 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2)^2 (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2)} - \\
& \frac{96 a f \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2)} - \\
& \frac{4 a^3 \pi^3 \operatorname{Sin}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{(a^2 + 4 f^2 \pi^2) (16 a^2 f^2 \pi^2 + (a^2 - 4(-1 + f^2) \pi^2)^2)} -
\end{aligned}$$

$$\begin{aligned}
& \left. \begin{aligned}
& \frac{16 a \pi^5 \operatorname{Sin}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{48 a f^2 \pi^5 \operatorname{Sin}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} \right) \Bigg/ \\
& \left(\begin{aligned}
& \frac{24 a^2 f \pi^3 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{32 f \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \\
& \frac{32 f^3 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{4 a^3 \pi^2 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{16 a \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \\
& \frac{48 a f^2 \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} \right)^2 + \\
& \left(- \frac{4 a^3 \pi^3 \operatorname{Cos}[f \pi] \operatorname{Cosh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \right. \\
& \frac{16 a \pi^5 \operatorname{Cos}[f \pi] \operatorname{Cosh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{48 a f^2 \pi^5 \operatorname{Cos}[f \pi] \operatorname{Cosh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \left. \left(4 a^3 \pi^2 \left(32 a^2 f \pi^2 - 16 f \pi^2 \left(a^2 - 4(-1 + f^2) \pi^2 \right) \right) \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi] \right) / \right. \\
& \left. \left(\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2 \right)^2 \right) \right)^2 \right) + \\
& \left(16 a \pi^4 \left(32 a^2 f \pi^2 - 16 f \pi^2 \left(a^2 - 4(-1 + f^2) \pi^2 \right) \right) \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi] \right) / \\
& \left. \left(\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2 \right)^2 \right) \right)^2 \right) - \\
& \left(48 a f^2 \pi^4 \left(32 a^2 f \pi^2 - 16 f \pi^2 \left(a^2 - 4(-1 + f^2) \pi^2 \right) \right) \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi] \right) / \\
& \left. \left(\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2 \right)^2 \right) \right)^2 \right) + \\
& \frac{32 a^3 f \pi^4 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right)^2 \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{128 a f \pi^6 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right)^2 \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} -
\end{aligned}
\right)
\end{aligned}$$

$$\begin{aligned}
& \frac{384 a f^3 \pi^6 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right)^2 \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{96 a f \pi^4 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \\
& \left(\frac{24 a^2 f \pi^3 \left(32 a^2 f \pi^2 - 16 f \pi^2 \left(a^2 - 4(-1 + f^2) \pi^2\right)\right) \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} \right) / \\
& \left(\frac{32 f \pi^5 \left(32 a^2 f \pi^2 - 16 f \pi^2 \left(a^2 - 4(-1 + f^2) \pi^2\right)\right) \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} \right) + \\
& \left(\frac{32 f^3 \pi^5 \left(32 a^2 f \pi^2 - 16 f \pi^2 \left(a^2 - 4(-1 + f^2) \pi^2\right)\right) \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} \right) - \\
& \frac{192 a^2 f^2 \pi^5 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right)^2 \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \\
& \frac{256 f^2 \pi^7 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right)^2 \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{256 f^4 \pi^7 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right)^2 \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{24 a^2 \pi^3 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{32 \pi^5 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \\
& \frac{96 f^2 \pi^5 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \\
& \frac{24 a^2 f \pi^4 \operatorname{Sin}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \\
& \frac{32 f \pi^6 \operatorname{Sin}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{32 f^3 \pi^6 \operatorname{Sin}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} \Bigg) / \\
& \left(\frac{24 a^2 f \pi^3 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \right. \\
& \left. \frac{32 f \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{32 f^3 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{4 a^3 \pi^2 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{16 a \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \\
& \frac{48 a f^2 \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} \Bigg) \Bigg/ \\
& \left(1 + \left(- \frac{4 a^3 \pi^2 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \right. \right. \\
& \frac{16 a \pi^4 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{48 a f^2 \pi^4 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{24 a^2 f \pi^3 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{32 f \pi^5 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \\
& \left. \frac{32 f^3 \pi^5 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} \right)^2 \Bigg/ \\
& \left(\frac{24 a^2 f \pi^3 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \right. \\
& \frac{32 f \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \\
& \frac{32 f^3 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right] \operatorname{Sin}[f \pi]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{4 a^3 \pi^2 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} + \\
& \frac{16 a \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} - \\
& \left. \frac{48 a f^2 \pi^4 \operatorname{Cos}[f \pi] \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2 + 4 f^2 \pi^2\right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4(-1 + f^2) \pi^2\right)^2\right)} \right)^2, \{f, \{0\}\}
\end{aligned}$$

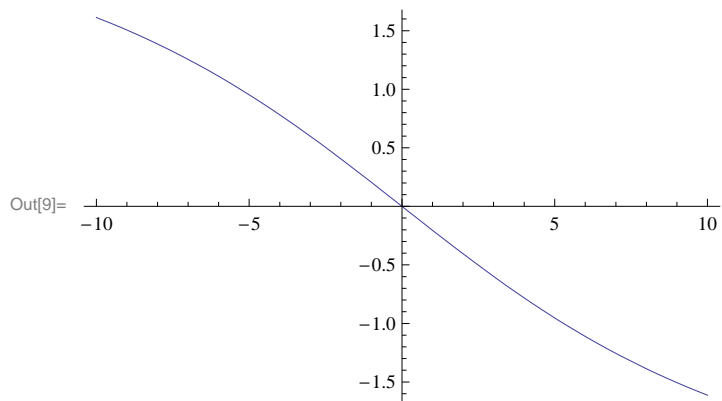
f 0

$$\text{Out}[7]= \frac{-\frac{4 a \pi^3 \operatorname{Cosh}\left[\frac{a}{2}\right]}{\left(a^2+4 \pi^2\right)^2}-\frac{16 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right]}{a\left(a^2+4 \pi^2\right)^2}+\frac{24 \pi^3 \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2+4 \pi^2\right)^2}+\frac{32 \pi^5 \operatorname{Sinh}\left[\frac{a}{2}\right]}{a^2\left(a^2+4 \pi^2\right)^2}}{\frac{4 a \pi^2 \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2+4 \pi^2\right)^2}+\frac{16 \pi^4 \operatorname{Sinh}\left[\frac{a}{2}\right]}{a\left(a^2+4 \pi^2\right)^2}}$$

$$\text{In}[8]= \text{FullSimplify}\left[\frac{-\frac{4 a \pi^3 \operatorname{Cosh}\left[\frac{a}{2}\right]}{\left(a^2+4 \pi^2\right)^2}-\frac{16 \pi^5 \operatorname{Cosh}\left[\frac{a}{2}\right]}{a\left(a^2+4 \pi^2\right)^2}+\frac{24 \pi^3 \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2+4 \pi^2\right)^2}+\frac{32 \pi^5 \operatorname{Sinh}\left[\frac{a}{2}\right]}{a^2\left(a^2+4 \pi^2\right)^2}}{\frac{4 a \pi^2 \operatorname{Sinh}\left[\frac{a}{2}\right]}{\left(a^2+4 \pi^2\right)^2}+\frac{16 \pi^4 \operatorname{Sinh}\left[\frac{a}{2}\right]}{a\left(a^2+4 \pi^2\right)^2}}\right]$$

$$\text{Out}[8]= \pi\left(\frac{2}{a}+\frac{4 a}{a^2+4 \pi^2}-\operatorname{Coth}\left[\frac{a}{2}\right]\right)$$

$$\text{In}[9]= \text{Plot}\left[\pi\left(\frac{2}{a}+\frac{4 a}{a^2+4 \pi^2}-\operatorname{Coth}\left[\frac{a}{2}\right]\right),\{a,-10,10\}\right]$$



$$\begin{aligned}
\text{In[11]} = & \text{FullSimplify} \left[\text{Sqrt} \left[\left(\left(- \frac{4 a^3 \pi^2 \text{Cosh} \left[\frac{a}{2} \right] \text{Sin} [f \pi]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4 (-1 + f^2) \pi^2 \right)^2 \right)} - \right. \right. \right. \\
& \frac{16 a \pi^4 \text{Cosh} \left[\frac{a}{2} \right] \text{Sin} [f \pi]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4 (-1 + f^2) \pi^2 \right)^2 \right)} + \\
& \frac{48 a f^2 \pi^4 \text{Cosh} \left[\frac{a}{2} \right] \text{Sin} [f \pi]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4 (-1 + f^2) \pi^2 \right)^2 \right)} + \\
& \frac{24 a^2 f \pi^3 \text{Cos} [f \pi] \text{Sinh} \left[\frac{a}{2} \right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4 (-1 + f^2) \pi^2 \right)^2 \right)} + \\
& \frac{32 f \pi^5 \text{Cos} [f \pi] \text{Sinh} \left[\frac{a}{2} \right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4 (-1 + f^2) \pi^2 \right)^2 \right)} - \\
& \left. \left. \left. \frac{32 f^3 \pi^5 \text{Cos} [f \pi] \text{Sinh} \left[\frac{a}{2} \right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4 (-1 + f^2) \pi^2 \right)^2 \right)} \right) \right)^2 \right] + \\
& \left(\left(\frac{24 a^2 f \pi^3 \text{Cosh} \left[\frac{a}{2} \right] \text{Sin} [f \pi]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4 (-1 + f^2) \pi^2 \right)^2 \right)} + \right. \right. \\
& \frac{32 f \pi^5 \text{Cosh} \left[\frac{a}{2} \right] \text{Sin} [f \pi]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4 (-1 + f^2) \pi^2 \right)^2 \right)} - \\
& \frac{32 f^3 \pi^5 \text{Cosh} \left[\frac{a}{2} \right] \text{Sin} [f \pi]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4 (-1 + f^2) \pi^2 \right)^2 \right)} + \\
& \frac{4 a^3 \pi^2 \text{Cos} [f \pi] \text{Sinh} \left[\frac{a}{2} \right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4 (-1 + f^2) \pi^2 \right)^2 \right)} + \\
& \frac{16 a \pi^4 \text{Cos} [f \pi] \text{Sinh} \left[\frac{a}{2} \right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4 (-1 + f^2) \pi^2 \right)^2 \right)} - \\
& \left. \left. \left. \frac{48 a f^2 \pi^4 \text{Cos} [f \pi] \text{Sinh} \left[\frac{a}{2} \right]}{\left(a^2 + 4 f^2 \pi^2 \right) \left(16 a^2 f^2 \pi^2 + \left(a^2 - 4 (-1 + f^2) \pi^2 \right)^2 \right)} \right) \right)^2 \right] \right] \\
\text{Out[11]} = & 2 \pi^2 \sqrt{\frac{-2 \text{Cos} [2 f \pi] + 2 \text{Cosh} [a]}{a^6 + 4 a^4 (2 + 3 f^2) \pi^2 + 16 a^2 (1 + 3 f^4) \pi^4 + 64 f^2 (-1 + f^2)^2 \pi^6}}
\end{aligned}$$

In[12]:= **Manipulate**[

$$\text{LogPlot}\left[2\sqrt{2}\pi^2\sqrt{\frac{-\text{Cos}[2f\pi] + \text{Cosh}[a]}{a^6 + 4a^4(2 + 3f^2)\pi^2 + 16a^2(1 + 3f^4)\pi^4 + 64f^2(-1 + f^2)^2\pi^6}}, \{f, -10, 10\}\right],$$

{a, 0, 10}]

Out[12]=

